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D-branes in massive IIA and solitons in Chern-Simons theory

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Abstract: We investigate D2-branes, D4-branes, and D6-branes parallel to D8-branes. The low energy world volume theory on the branes is non-supersymmetric Chern-Simons theory. We indentify the fundamental strings as the anyons of the 2+1 Chern-Simons theory and the D0-branes as solitons. The Chern-Simons theory with a boundary is modeled using a brane box made from NS5-brane. The brane set-up provides for a graphical description of anomaly inflow. We also model the 4+1 Chern-Simons theory using branes and conjecture that D4-branes with a boundary describes a supersymmetric version of Kaplan's theory of chiral fermions.

Keywords: Solitons Monopoles and Instantons, Chern-Simons Theories, Brane Dynamics in Gauge Theories, Anomalies in Field and String Theories.

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1. Introduction

Modeling other physical systems using D-branes is a recent trend in string theory: black holes, supersymmetric gauge theories, non-commutative field theory, the fractional quantum Hall effect, and even the Standard Model are all examples of systems that have string theory realizations. It has been known from sometime that Dp-branes in massive IIA supergravity have a Chern-Simons term

$$\mathcal{L}_{\mathrm{D}p} = kA \wedge F^{p/2} \tag{1.1}$$

on their world volume [1] enabling one to model Chern-Simons theories in string theory. In this paper we will explore the consequences of having Dp'-branes in the Dp-branes in massive IIA and take up the task of modeling anyons, solitons, and edges in the Chern-Simons theories on the brane world volume theory.

Chern-Simons theories are very interesting for many reasons: one is that electrically charged particles in the Chern-Simons theory can have fractional statistics: they are neither bosons nor fermions, they are anyons. Anyons have important roles to play in theories of the fraction quantum Hall effect, HiTC superconductivity, and more recently in quantum computing. The Chern-Simons term itself is related to gauge anomalies in one higher dimension. It follows from this that at low energies where one can ignore the kinetic term the 2+1 Chern-Simons theory is topological. It is an example of an exactly protected quantity that does not depend on supersymmetry. The 4+1 Chern-Simons theory on a boundary is interesting because it gives the domain wall fermions studied by Kaplan [2]. Perhaps if we want to solve QCD using string theory, we should include chiral symmetry in the way suggested by Kaplan.

In this paper, for a D2-brane in the presence of a D8-brane, we will identify the 8-2 string ends on the D2-brane as the anyons of the 2+1 non-supersymmetric Chern-Simons theory and D0-branes outside the D2-branes with k strings attached as non-topological solitons with electric charge k. By introducing NS5-branes we can add a boundary to our D2-brane and study the chiral theory on the boundary. We identify the massless chiral fermions as the light strings on the 2+1 boundary. This provides for a microscopic picture of charge inflow: long strings move to the boundary creating massless charged particles from "nowhere" and therefore violating charge conservation in the 1+1 dimensional theory. For a D4-brane the situation of a D8-D4-D0 is supersymmetric. The D0-brane is an instanton carrying k units of electric charge in the D4-brane world volume. Putting a boundary on the D4-brane induces chiral fermions in 3+1 and reminds one of Kaplan's theory of chiral symmetry on the lattice [2, 3].

The outline of the paper is as follows: in section 2, we will review the string creation process which happens when a D0-brane crosses a D8-brane and go over the argument of [4] as to why this is related to an anomaly inflow of a 1+1 dimensions.

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In section 3, we will relate the anomaly inflow of a 3+1 gauge theory to a string creation when a D2-brane crosses a D8-brane carrying magnetic flux. The magnetic flux is produced by a D0-brane inside the D2-brane. We identify the k strings created as k anyons of the 2+1 Chern-Simons theory and the D0-brane as a soliton carrying $-k$ units of electric charge. In section 3.8 we propose that a U(M) level k non-abelian Chern-Simons theory is dual to the fractional quantum Hall effect at level $\nu = k/M$. This proposal is based a T-duality relation between the brane scenario discussed in this section and the brane scenario of [5]. In section 3.9, we show how to put the Chern-Simons theory on a boundary by introducing NS5-brane on the edge of the D2-brane. We then show how anomaly inflow from the 2+1 theory into the chiral theory on the 1+1 boundary is realized in this string model. In section 4, we generalize the results of the 2+1 dimensional Chern-Simons theory to 4+1 dimensions and propose that they are related to the domain wall fermion proposal used in lattice QCD [2, 3].

There are several papers on supersymmetric Chern-Simons theories in 2+1 realized on branes of type-IIB [6, 7, 8, 9] as well as papers on solitons [10]. The novelty here is that the Chern-Simons coefficient has as different origin; it comes from the D8-branes. Moreover, the Chern-Simons theory on the boundary in easily studied in this brane construction, it is related to string creation of [11], and this construction has a close relationship with the fractional quantum Hall construction of [5] as will be discussed.

For other occurences of Chern-Simons theory in string theory see [12, 13, 14].

2. String creation by a D0-brane crossing a D8-brane: a review of the supersymmetric case

2.1 A few comments about massive IIA supergravity

Massive IIA has a positive cosmological constant proportional to $(F^{(0)})^2$, the dual of the ten form field strength that couples to the D8-brane, and a linear dilaton potential[15]. There is no known ulta-violet definition of massive IIA supergravity. The problem is that the string coupling constant grows stronger the farther one is away from the D8-brane

$$\frac{1}{g_s} = \frac{1}{g_0} - |r - r_0|^{5/4}\,. \tag{2.1}$$

At $r = r_0$, $g_s = g_0$ and then grows until $r = r_0 + (1/g_0)^{4/5}$ where the coupling blows up. However we can make this distance (measured in string units) very big by taking g_0 to be very small. Then we can work in the supergravity regime near the D8-brane and not venture out to where gravitational effects become strong.

If one doesn't feel comfortable with the Massive IIA supergravity solution one could introduce orientifold planes which will cut off the growth of the dilaton before one looses control. One should just make sure to arrange the probe brane (i.e. D0,D2,D4-brane) such that there are unequal numbers of D8-branes one the left and right hand side of the probe. This will insure that the Chern-Simons coefficient does not vanish since as we shall see

$$\frac{\tilde{k}}{2} = \frac{k-16}{2}, \tag{2.2}$$

where k is the number of D8-branes and where the -16 comes from the orientifold plane charge. In such a theory the coefficient of the Chern-Simons theory on the probe brane $\tilde{k}$ is always integer.

Massive IIA supergravity is called "massive" because the antisymmetric tensor field $B_{\mu\nu}$ gets a mass by eating the RR-vector field C_μ. One can understand this by writing a new gauge invariant RR field strength $\tilde{G}$ as

$$\tilde{G} = G + kB\,, \tag{2.3}$$

where $G = dC$. Expanding out the kinetic term for C in the IIA action, we find a mass term for B

$$\mathcal{L}_{IIA} = \tilde{G} * \tilde{G} = G * G + kB * G + k^2 B * B \tag{2.4}$$

as well as a coupling of B to dC.

2.2 Anomaly in 1+1 dimensions

Let us consider two D5-branes on type-IIB string theory extending in directions 012345 and 016789. As described in [4], the low energy theory on the two dimensional intersection is a U(1) $\times$ U(1) field theory with $N = (0,8)$ supersymmetry. There is a chiral spinor charged as $(1,-1)$ under the gauge group which is the 5-5 string. Notice that one cannot give a mass to the chiral fermion since in the brane set-up that involves separating the D5-branes and stretching the 5-5 string and there is no such direction where this is possible. This theory has an anomaly at one loop if one turns on a background gauge field strength F_{01} since the current obeys the relation

$$\partial^\sigma J^3_\sigma = F_{\mu\nu}\epsilon^{\mu\nu} \tag{2.5}$$

and is therefore not conserved when $F_{01} \neq 0$. Because the 1+1 dimensional gauge theory is anomalous but the ten dimensional bulk theory is anomaly free, there must be inflow of charge that cancels the anomaly and restores gauge invariance. This inflow comes from the D5-branes. As was noted in [16] the inflow current is perpendicular to the electric field. This is reminiscent of the quantum hall effect which we shall make more precise later in section 3.8.

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2.3 Chern-Simons theory in 0+1 dimensions

Upon T-duality along 12345, the theory discussed in section 2.2 becomes a D0-brane and a D8-brane. There is a U(1) gauge symmetry on the D0-brane and a U(1) flavor symmetry on the D8-brane since the 8-brane coupling is very weak in the infra-red. Virtual 0-8 strings are fermions with a real mass $\langle\phi\rangle = m$, the separation distance between the 0-brane and the 8-brane. Doing a 1-loop calculation and integrating out the massive fermions induces a Cherm-Simons term and a potential for the scalar ϕ.

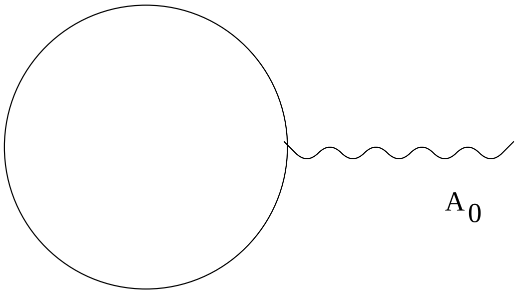

Figure 1: Integrating out the fermions in 0+1 induces a Chern-Simons term.

$$\mathcal{L}_{D0} = -\frac{1}{2}\frac{m}{|m|}(A_0 + \phi)\,. \tag{2.6}$$

The 1-loop term in the open-string channel is dual to a tree-level term in the closed string channel. Due to a non-renormalization theorem potential is the same whether calculated in supergravity or in field theory [17, 18, 19]. In fact we can do the supergravity calculation as follows: the metric for a Dp'-brane is given by

$$ds^2 = f(r)^{-1/2}dx^2 + f(r)^{1/2}dy^2\,, \tag{2.7}$$

where x^μ $\mu = 0, \ldots, p'$ are the coordinates parallel to the brane, y^a $a = p'+1, \ldots, 9$ are the coordinates perpendicular to the brane, and $r = \sqrt{y^a y^a}$. The dilaton obeys the equation

$$e^{-2\phi} = f^{\frac{3-p'}{2}}\,. \tag{2.8}$$

A parallel probe Dp-brane action is

$$S = e^{-\phi}\int\sqrt{\det G}\,, \tag{2.9}$$

where $G_{\mu\nu}$ is the metric on the Dp-brane induced by the metric in the bulk (2.7). Plugging (2.7) and (2.8) into (2.9) we find that the potential for the scalar field on the brane is

$$V(\phi) = f^{\frac{p'-p-4}{4}}(\phi)\,, \tag{2.10}$$

where the scalar field is related to the coordinates via the relation $\phi = M_s^2 x$. For a D8-brane and a D0-brane we find that

$$V(\phi) = f(\phi) = -\phi\,. \tag{2.11}$$

Notice that there is a difference in a factor of 1/2 between equation (2.11) and (2.6). This is because of a different normaliztion conversion: in supergravity it is assumed that the value of $F^{(0)}$ jumps from 0 to 1, but in field theory it is assumed that there is a jump from $-1/2$ to 1/2. This is related to the notion of a "half-string creation" — see [20] for discussion.

As the D0-brane crosses the D8-brane the chirality of the fundamental field changes sign. Therefore (2.6) also changes sign. To prevent such a discontinuous jump, we must introduce some charge that can compensate for this transition. This is supplied by the fundamental string that is created. In fact, the electric flux that one must turn on in the 1+1 theory to induce an anomaly becomes momentum of the D0-brane transverse to the D8-branes [4].

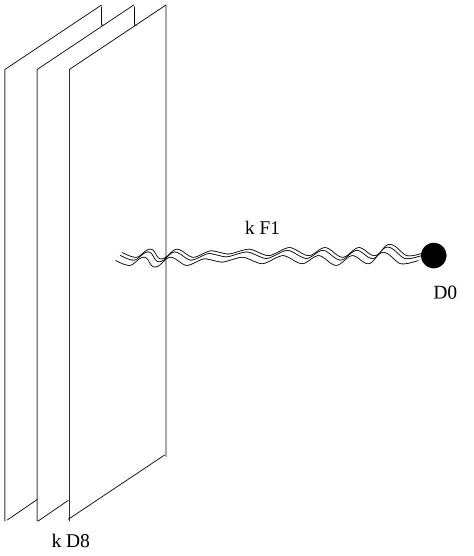

Figure 2: When a D0-brane crosses k D8-branes, k fundamental strings are created.

2.4 Charge conservation in ten dimensions

Another way one can view the brane creation is through supergravity [21, 22]. Consider the action for the antisymmetric field which couples to the string.

$$\mathcal{L}_{IIA} = H \wedge *H + B^{(2)}\delta^{(8)} + kB^{(2)}G^{(8)}_{RR}\,. \tag{2.12}$$

Where the delta function is the string souce and the 8-form field strength couples magnetically to the D0-brane. The coupling of the antisymmetric tensor field to the 8-form field strength is peculiar to massive IIA supergravity [23]. The equations of motion following from this action are

$$d * H + \delta^{(8)} + kG^{(8)}_{RR} = 0\,. \tag{2.13}$$

Because flux lines have no where to go on a sphere

$$\int_{S^8} d * H = 0 = \int_{S^8} (\delta^{(8)} + kG^{(8)}_{RR}) = (Q_{NS} + kQ_{DO})\,. \tag{2.14}$$

If we didn't have the coupling in (2.12) charge conservation would have been violated. This is the reason that a single string can end on a D0-brane only in massive IIA supergravity and not in ordinary type IIA [21].

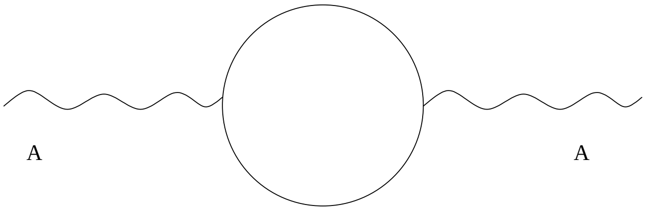

Figure 3: Integrating out fermions induces a Chern-Simons term in 2+1 dimensions.

3. String creation by a D2-brane crossing a D8-brane: the non-supersymmetric case

3.1 Anomaly in 3+1 dimensions

Now let's consider two D6-branes in type-IIA string theory along directions 0123456 and 0123789. The theory on the intersection is a 3+1 dimensional $\mathrm{U}(1) \times \mathrm{U}(1)$ gauge theory with a single chiral fermion with charge $(1, -1)$ under the gauge group. In terms of branes the chiral fermion comes from the 6-6 strings. Again one cannot give the chiral fermion a mass in the brane picture since one cannot separate the 6-branes. This theory is non-supersymmetric, and one can think of it as coming from a supersymmetric theory with a FI D-term [24, 25]. There is a gauge anomaly from the one-loop triangle diagram which is cancelled by inflow from the 6+1 dimensional theory off the intersection. However, according to

$$\partial^\sigma J^5_\sigma = \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} \tag{3.1}$$

there will be no anomaly unless there is a background field where $F \wedge F$ is non-zero. To satisfy this we must have non-zero electric field F_{01} and non-zero magnetic field F_{23}. In terms of branes the electric flux is a fundamental string in directions 03 bound to the D6-brane while the magentic flux is a D4-brane in directions 03456 parallel to and bound to the D6-brane in directions 0123456.

Another way to induce an anomaly in 3+1 dimensions is to put an instanton in the 3+1 theory. Such an instanton could be provided by placing a euclidean D2-brane inside the D6-brane along spatial directions 456.

3.2 Chern-Simons theory in 2+1 dimensions

Now let's T-dualize the brane configuration along directions 3456 and in doing so dimensionally reduce the theory on the intersection to 2+1.[1] The system is now a D2-brane along 012 and a D8-brane along 012456789. In addition to the usual fields of $N = 8$ super Yang-Mills in 2+1 dimensions, the gauge field, seven scalars, and eight

[1]One could equivalently consider two D5-branes intersecting over a 2+1 surface in IIB.

fermions all in the adjoint representation, there is a parity violating fundamental fermion that couples to the real adjoint scalar corresponding to the 3-direction.

$$\mathcal{L}_{D2} = X\bar{\psi}\psi + \cdots . \tag{3.2}$$

Setting $\langle X \rangle = m$, at one loop, after integrating out the massive fermion there is a Chern-Simons term induced.

$$\mathcal{L}_{D2} = \frac{1}{2}\frac{m}{|m|}\epsilon^{\mu\nu\rho}A_\mu F_{\nu\rho} . \tag{3.3}$$

This one-loop term is not renormalized [16] and therefore we conjecture it to be the same as the term calculated using supergravity. k D8-branes give a Chern-Simons coefficient $k/2$. The non-renormaliztion of the coefficient of (3.3) is related to D8-brane charge conservation in the bulk theory. Interestingly, as was mentioned in section 2.1 in a IIA theory with O8-planes as well as D8-branes the Chern-Simons coefficient is always integer. This agrees with the fact that there is an anomaly in the compact U(1) field theory with even Chern-Simons coefficient [26].

There is a different 1-loop diagram that contributes to the Coleman-Weinberg potential. For $X \ll M_s$ it is

$$V(X)_{FT} = -|X|^2 . \tag{3.4}$$

In the other limit, $X \gg M_s$ one cannot use field theory because all of the stringy modes become relevant, but one can, by channel duality, use supergravity. Using the formulas (2.7), (2.8) and (2.9) in the previous section, one finds that the potential for the 2+1 dimensional theory on the D2-brane is

$$V(X)_{\mathrm{SUGRA}} = kf^{1/2} = \frac{1}{g_s}M_s^3(1 - g_s k M_s X)^{1/2} = \frac{M_s^3}{g_s} - kM_s^4 X + \cdots \tag{3.5}$$

for small values of g_s. k is the number of D8-branes. The low energy theory on the N_c D2-branes is then a SU(N_c) gauge theory with 8 gauginos and 6 scalars all transforming in the adjoint representation.

3.3 D0-branes in D2-branes with D8-branes

Now let's consider putting a D0-brane inside the D2-brane in massive IIA.[2] We will not attach strings to the D0-brane and we will see in the next section that there is no violation of charge conservation. The potential $C^{(1)}_{RR}$ that couples to the D0-brane couples also to the gauge field on the brane F through the coupling

$$\mathcal{L}_{D2} = C^{(1)} \wedge F^{(2)} . \tag{3.6}$$

[2]In much of this discussion we will imagine holding the D2-brane fixed. This is accomplished with NS5-branes and will be discussed in section 3.8.

Therefore, the D0-brane charge appears in the 2+1 world volume as magnetic flux. Moreover, because of the Chern-Simons term in the Maxwell lagrangian on the D2-brane (3.3) we see that the magnetic flux induces k units[3] of electric charge

$$d * F = kF\,. \tag{3.7}$$

From string theory, typically one expects that the D0-brane dissolves inside the D2-brane. This is because in IIA string theory there is an instability due to a tachyonic mode on the 0-2 string. However, we will argue that this tachyon does not have to roll off to infinity, but can be in fact stabilized by the 1-loop term coming from integrating out the 0-8 strings which induce the linear scalar potential, the superpartner of the Chern-Simons term (2.6). The potential for the D0-brane is

$$V_{D0} = -k\phi + \phi^2 q^2 - \mu^2 q^2 + \cdots\,, \tag{3.8}$$

where m is the tachyonic mass of the 0-2 string. The equations of motion following from (3.8) are

$$\begin{aligned} -k + \phi q^2 &= 0 \\ (\phi^2 - \mu^2) q &= 0\,. \end{aligned} \tag{3.9}$$

Equations (3.9) have two minima: one at $q = 0$ with $\phi = \infty$ the other at $q = \infty$ with $\phi = 0$. The solution with $q = \infty$ corresponds to the D0-brane spreading out while $q = 0$ corresponds to a confined D0-brane. Physically the latter solution corresponds to the D0-brane being pushed away from the D8-brane by the linear potential. However, the D0-brane cannot leave the D2-brane or it will violate charge conservation. The D0-brane is stuck to the D2-brane but repelled from the D8-brane. This causes the D2-brane to bend. The scalar field on the D2-brane satisfies Laplace's equation

$$\nabla^2 X = \delta^{(2)}(x)\,. \tag{3.10}$$

This equation has a logarythmic solution in 2+1 dimensions. These vortices are therefore charged under the electric field, the Higgs field, and carry one unity of magnetic flux. There are three forces now on the D0-brane: repulsion due to the D8-brane, attraction due to the bending of the D2-brane, plus an attraction due to the D2-brane. When the D0-branes is far from the D2-brane, the attractive bending cancels the D8-brane repulsion, and so the D2-brane attraction wins. When the D0-brane is inside the D2-brane, the D8-brane repulsion dominates since there is no

[3] Here we must be careful to note that we have renormalized k such that the Chern-Simons coefficient is twice the eightbrane charge.

bending and the D0-brane cannot feel the D2-brane force. We claim then that the forces balance when the D0-brane is a string length from the D2-brane.[4]

3.4 Non-topological solitons

Its clear that the D0-branes are non-topological solitons of the Chern-Simons theory. Topological vortices are typically associated with a broken U(1) gauge invariance, but here the gauge symmetry is preserved. If fact there are no massless fundamental scalar fields with which we could Higgs the gauge field. There are however in Chern-Simons theories stable non-topological solitons which are like the Q-balls of [28]. In most gauge theories flux wants to spread out since spreading lowers the energy. One can see this from the formula for the energy

$$\mathcal{E} = V(E^2 + B^2)\,, \tag{3.11}$$

where V is the volume of the soliton and E is the electric field and B is the magnetic field. If we take the flux to be constant

$$\Phi = BV = N \tag{3.12}$$

then the equation for the energy becomes

$$\mathcal{E} = \frac{N^2}{V}\,. \tag{3.13}$$

Therefore the more spread out the object, the lower its energy. This is what one is familiar with in string theory: the D0-brane wants to spread out in the D2-brane. This will not be the case if the soliton is also charged under some scalar field since the energy of the scalar field will grow like the volume of the soliton. Therefore once the D0-brane is charged under the neutral scalar field X, its size will be stabilized. The neutral scalar field corresponds to motion of the D2-brane transverse to the D8-brane which we can make massive by introducing more branes (as will be explained later in section 3.9). Presumably X will have a solution such as

$$X(r) = \log r \tag{3.14}$$

since it satisfies (3.10). Clearly this solution does not have finite energy since X does not approach zero at $r = \infty$ where r is the spatial coordinate on the D2-brane. However, the total system does have finite energy because there are strings in the

[4]It is interesting to note that the mechanism of relocalizing the D0-brane inside the D2-brane can be understood as a 1-loop effect in 2+1 field theory: there first are massive fermions and a magnetic flux, then we integrate out the fermions which generates a Chern-Simons term giving the gauge field a mass, the magnetic flux then confines into a magnetic vortex. This scenario is similar to the confinement by instantons mechanism proposed in [27]. There euclidean D0-branes were argued to confine strings inside D4-branes.

theory charged oppositely under the scalar field and so the total scalar charge is zero just like the total electric charge is zero. The logarythmic solution (3.14) will turn into multipole solution which has a power law form and dies away at infinity. In this way, the non-topological solitons that we find in this string theory set-up appear to be different from the non-topological solitons discussed in the literature since there a single soliton has finite energy [29, 30].

3.5 Topological solitons

Since we have discovered that we can have non-topological solitons in our brane model of the Chern-Simons theory, it is natural to ask whether we can have topological solitons as well. To have topological solitons we must introduce some extra branes that will play the role of the fundamental scalar field q. We will choose to add D4-branes.[5] The boundary conditions on the 2-4 strings allow for one tachyonic scalar field when the D2-branes are on top of the D4-branes, and what's more we already know that the neutral adjoint scalar field has a tadpole from (3.5). The potential for the D2-brane can then be modeled as we did for the D0-brane (3.8) by the following equation

$$V_{D2} = -\mu^2 q^2 - kX + X^2 q^2 \,. \tag{3.15}$$

This solution again has the following minima: if q gets a vacuum expectation value and $X = 0$, the the U(1) gets broken and there can be topological solitons. In the brane picture the D2-brane will dissolve into the D4-brane while the D0-branes remain of finite size since they are instantons in the D4-branes. On the other hand another minimum is when $q = 0$ and X gets a vacuum expectation value. This will lead to the non-topological solitons that we discussed above. In the brane picture, the D8-brane pushing on the D2-D4 bound state will cause the D4-brane to bend while the D8-brane pushing on the D2-D0 bound state will cause the D2-brane to bend. Both the D2-brane and the D0-brane will be undissolved.

3.6 Charge conservation in ten dimensions with a D2-brane

As in section 2.4 we can look at the equations of motion following from the massive IIA action and see how charge conservation is satisfied. The action is the same as (2.12) except for a new coupling of the B-field to the gauge field on the brane F.

$$\mathcal{L}_{IIA} = H \wedge *H + B^{(2)}\delta^{(8)} + kB^{(2)}G^{(8)}_{RR} + B^{(2)} \wedge *F^{(1)}\delta^{(7)} \,. \tag{3.16}$$

The equations of motion are

$$d * H + \delta^{(8)} + kG^{(8)}_{RR} + *F^{(1)}\delta^{(7)} = 0 \,. \tag{3.17}$$

[5] We could have also used D6-branes but that adds some complications in massive IIA as we will see later.

Now when we integrate over the 8-sphere we find

$$Q_{NS} + kQ_{D0} + \int_{S^1} *F = 0\,. \tag{3.18}$$

Notice that this equation can be satisfied in many different ways:

1. The D0-brane charge can cancel the electric charge on the brane. This solution is a single vortex in 2+1 dimensions.

$$kQ_{D0} + \int_{S^1} *F = 0\,. \tag{3.19}$$

2. The string charge can cancel the electric charge on the brane. This solution is an electron in 2+1 dimensions.

$$Q_{NS} + \int_{S^1} *F = 0\,. \tag{3.20}$$

3. The D0-brane charge cancels with the D2-brane charge. There is no electric charge in the 2+1 world volume.

$$Q_{NS} + kQ_{D0} = 0\,. \tag{3.21}$$

We conclude that a D0-brane in a D2-brane in massive IIA behaves like a cut string. The cut string is sensible solution if it has its end on a D2-brane, but not is if has its end in the bulk: likewise, the D0-brane does not violate charge conservation if it is inside a D2-brane in massive IIA, but it cannot go outside of the D2-brane.

3.7 Non-supersymmetric String creation

Because the real mass of the fundamental field changes sign from one side of the D8-brane to the other, the sign of (3.3) also changes sign. However, if there is no background field strength $F_{\mu\nu}$, then (3.3) is zero, and there is no problem. However, if $F_{\mu\nu}$ is non-zero, then there is a problem with charge conservation because equation (3.7) say that the D0-brane carried k units of electric charge when on one side of the D8-brane but $-k$ units of electric charge on the other side. The difference in electric charge must be compensated for by a string creation. One way to have a non-zero field strength $F_{\mu\nu}$ on the brane is, as we saw in section 3.3 , to bind a D0-brane to the D2-brane. The D0-brane is exactly T-dual to the D4-brane from the previous section 3.1 which played the role of magnetic flux inside the D6-brane. When the D2-D0 system crosses the D8-brane, a string is created. Notice that the momentum of the D2-D0 bound state in the 3-direction is T-dual to the electric field that was necessary to have an anomaly in 3+1 dimensions.

In the previous section we said that we could induce charge inflow into the anomalous 3+1 theory by adding an instanton. Now we will show that upon dimensional reduction to 2+1 dimensions the instanton implies charge inflow. T-dualizing the D6-D6 configuration with a euclidean D2-brane along directions 3456 we find a D2-D8 configuration with a euclidean D0-brane along the 3-direction. How do we think about a euclidean D0-brane? The analogy comes from open-closed string duality in string theory. There events such as the creation and annihilation of charged particles, i.e. loops are represented in the closed string channel as a propagating particle. However we are use to thinking of a propagator as a particle with some momentum moving between two sources. So let think of the euclidean D0-brane in the same way. The euclidean D0-brane is a D0-brane that moves along the 3-direction. Since there is an attractive force between the D2-brane and the D0-brane, they will form a bound state and move together. Therefore we have a D2-brane with a D0-brane bound to it, moving in the 3-direction just as discribed above. When the D2-brane crosses the D8-brane a string is created. This is the T-dual of the charge inflow in 3+1 dimensional anomalous theories.

3.8 Strings as anyons

In the previous section 3.2 we saw that integrating out fermions in 2+1 QED leads to a Chern-Simons term. Because of the equations of motion (3.7), whenever we have one unit of magnetic flux we have k units of electric charge. Conversely when we have 1 unit of electric charge we can have $1/k$ units of magnetic flux. That is fractional magnetic flux. Moreover when one circles such fractional magnetic flux with an electrically charged particle on picks up a fractional phase due to the Bohm-Ahramov effect [31].

$$e^{2\pi i e\int A\cdot dx} = e^{2\pi i\frac{e^2}{k}}\,. \tag{3.22}$$

These are the anyons of condensed matter physics with spin $1/k$. In the brane set-up the ends of the strings are electrically charged. According to the Chern-Simmons equations of motion this implies that each string end should carry $1/k$ units of magnetic flux. If the D0-brane crosses k D8-branes it will have k strings ending on it. Therefore, a D0-brane and k strings have exactly opposite charge and flux. The strings are the anyons and the D0-branes have the quantum numbers of a composite of k anti-anyons. In fact, if one begins with the D0-brane to the D8-brane-side of the D2-brane and pulls the D0-brane through the D2-brane, one will have k strings on either side of the D2-brane. If one now moves the D0-brane down, the strings on the D8-brane side can all move independently while the strings on the D0-brane side are all bound to the D0-brane. In this way, the soliton is literally a composite of k anyons.

3.9 Chern-Simons as dual to the fractional quantum Hall effect

In [5] a brane model for the fractional quantum Hall effect involving M D0-branes, N D2-branes and k D6-branes was presented. The system of N D0-branes, M D2-

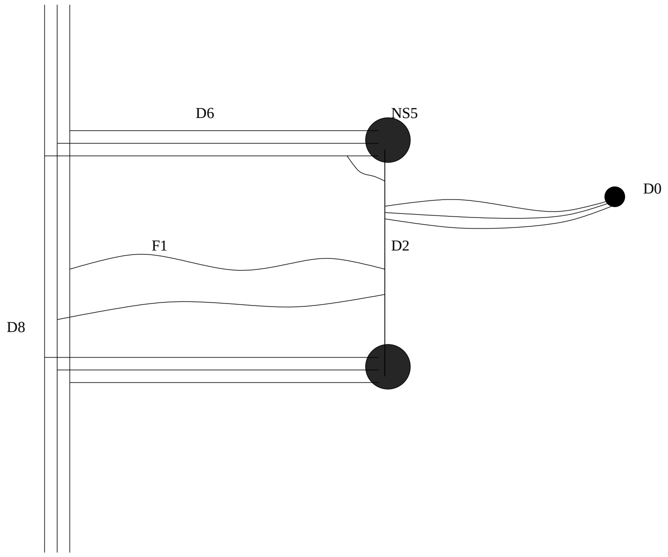

Figure 4: Long 8-2 strings want to move to the boundary, turn into 6-2 strings and become light. This provides the current which flows into the 1+1 theory on the boundary of the Chern-Simons theory making it anomalous. A D0-brane outside the D2-brane has k strings ending on it. The D0-brane behaves like a soliton with electric charge k and magnetic flux 1. The string ends on the D2-brane behave like anyons of electric charge 1 and magnetic flux $1/k$.

branes, and k D8-branes presented in this paper is T-dual along the spatial directions of the D2-branes to the configuration of [5]. In terms of the low energy effective field theory this duality implies that an fractional quantum Hall effect with filling factor $\nu = k/M$ is dual to a U(M) non-abelian theory with Chern-Simons level k. One should think of the D2-D0-D6-brane theory on a T^2 rather than the S^2 of [5]. If, for example, the filling factor were 1/3, then there is one D6-brane, one string, and 3 D0-branes per cell defined by the periodic torus. This torus here is playing the role of the periodic lattice in condensed matter systems. This is then T-dual to a U(3) theory on a similar periodic lattice with the D0-branes playing the role of the compensating negative charge. The atomic nuclei in condensed matter systems.

It is amusing that this relates the FQHE to a non-abelian Chern-Simons theory where the electrons carry a non-abelian index. This suggests that in this picture the electron has structure much as in composite fermion proposals of [32].

3.10 Chern-Simons theory on a boundary

One can make a brane set-up to model the Chern-Simons theory on a boundary. We simply have the D2-brane end on NS5-branes. Consider D2-brane in 012, D8-branes in 012456789 as before. Now add NS5-branes along 014567. The NS5-brane create a boundary on which the D2-brane can end. However, as with a D0-brane in massive

IIA the coupling in equation (2.12) demands that a D6-brane end on the NS5-branes to conserve NS charge. Therefore, along the boundary of the D2-brane there will be D6-branes in the 0134567 directions. The massless 2-6 strings are in fact chiral fermions because the boundary conditions for a 2-6 strings is still 6ND, the same boundary conditions as for the 2-8 strings; the strings can move from the 8-brane onto the 6-brane without changing their boundary conditions. We know that the 2-8 strings are parity violating in the 2+1 theory since they come from a dimensional reduction of the chiral 3+1 theory discussed in section 3.1.

The chiral theory on the right edge of the D2-brane carries k right moving massless chiral fermions while on the left edge there are k left-moving chiral fermions. If the boundary were a circle, then the right movers would go around and turn into left movers. Note that the left and right moving 2-6 strings *cannot* join to make a 6-6 string that is neutral under the U(1) gauge field on the D2-brane. However, a left-moving 2-6 string can join with a anti-right-moving 2-6 string. Such a joining resembles then confinement of a quark-anti-quark pair and will be discussed in a later section in detail.

The 2+1 Chern-Simons theory is not gauge invariant on the boundary. One can see this by starting from the lagrangian

$$\mathcal{L}_{D2} = kA_2F_{01} + J^2A_2 + \cdots . \tag{3.23}$$

If we now take consider a small variation in A_2

$$\delta A_2 = \partial_2\alpha . \tag{3.24}$$

Under this variation we find a surface term

$$\alpha(kF_{01} + J^2) . \tag{3.25}$$

For this surface term to vanish means that

$$kE_1 = J^2 , \tag{3.26}$$

which states that electric field along the boundary is proportional to current moving transverse to the field in the bulk. This is very much like the Hall effect. Moreover, equation (3.26) is just the anomaly equation in 1+1 (2.5) if we identify

$$J^2 = \partial_0 K^{02} , \tag{3.27}$$

where $K^{\mu 2}$ is the "axial" current in 1+1 theory. This suggests that there should be a 1+1 gauge theory with k chiral fermions on the boundary.

How is the 1+1 anomaly equation (3.26) manifest in the brane construction? The massive 8-2 strings in the bulk move to the boundary where they become massless 6-2 strings which then begin to move along the boundary at the speed of light. From the

1+1 point of view it appears that charge was created from nowhere and that there is an anomaly. However, from the 2+1 point of view charge entered the boundary from the bulk and there is no violation of charge and no anomaly. This is a microscopic picture in terms of strings of the inflow mechanism of [16].

Another attractive feature of having a boundary for the D2-brane is that this lifts the scalar field with the runaway potential (3.5). Therefore, the 2+1 field theory is stabilized, at least perturbatively. Likewise, the supergravity theory is stabilized for $g_s \ll 1$. This is very similar to the set-up discussed in [33].

3.11 Chiral anomaly on the boundary dimensionally reduced to 0+1

We can consider dimensionally reducing the above brane configuration describing a 2+1 Chern-Simons theory with a boundary. This amounts to T-dualizing the branes along the 1-direction. The D2-brane becomes a D1-brane extending in the 2-direction suspended between the two NS5-branes. The anomalous boundary theory is then 0+1 dimensional Chern-Simons. The D6-brane ending on the NS5-brane turns into a D5-brane extending along directions 034567 which ends on the NS5-brane in directions 014567. The 5-branes then join into a three 5-brane junction involving an D5, NS5, and $(1,1)$ 5-brane extending in 45 degrees along the 13-direction. The D1-brane can now move along the NS5-brane. When the D1-brane encounters the three 5-brane junction it can now move onto a different branch. As it crosses onto the $(1,1)$ 5-brane it will must turn into a $(1,1)$ 1-brane. This means a fundamental string must be appear. The inflow of the current flowing into the 1+1 boundary theory maps to the D1-string turning into a $(1,1)$-string under T-duality.

4. D0-D4-D8. Five dimensional Chern-Simons theory

Consider a system of D4-branes along directions 01245, D8-branes along direction 012456789, and D0-branes marginally bound to the D4-brane. This theory is similar to the case with a D0-D2 bound state in the background of a D8-brane, except that it is supersymmetric. In this case which was analyzed in [4], the D0-brane is an instanton in a 4+1 dimensional worldvolume theory. Due to the D8-brane there is a 5d Chern-Simons coupling

$$\mathcal{L}_{D4} = F \wedge *F + kA \wedge F \wedge F\,. \tag{4.1}$$

Since both terns in the action (4.1) have two derivatives we cannot ignore the kinetic term as we did in the 2+1 Chern-Simons theory.

If $F \wedge F$ is non-zero and the D4-brane crosses the D8-branes, a string must be created to maintain charge conservation on the D4-brane. Because of the Chern-Simon term (4.1) the instanton carries electric charge. What happens when the instanton gets fat? We'll conjecture that the instantons cannot get fat in 4+1 Chern-Simons theory. From the 0+1 gauge theory point of view this means that the Higgs branch is lifted. The way that this happens is similar to the way magnetic flux became confined in the previous section. Separating the 8-0 strings from the D0-brane on putting them far away on the D4-brane for the moment, from the point of view of the $0+1$ dimensional theory living on the D0-brane there is the following potential:

$$V_{D0} = k\phi + \phi^2 q^2 + \cdots , \tag{4.2}$$

where we have only written the terms involving ϕ. The minimum solution to (4.2) is $q = 0$ and $\phi = -\infty$. Because of the linear potential for ϕ, the Higgs branch of the 0+1 dimensional theory is lifted. This says that, the D0-branes want to shrink to zero size inside the D4-brane and leave but it cannot since it carries electric charge. So the D4-brane bends in response to the force in the D0-brane due to the D8-brane. One can picture this as the D0-brane pulling off little one dimensional pieces of the D4-brane. However, as was shown in [34] these pieces are just fundamental strings. The final picture is then the D0-brane outside the D4-brane with k strings attached. From the instanton moduli space point of view, the D0-D8 force breaks the conformal invariance of R^4 and so the instanton size is no longer a moduli. With the 8-0 strings attached to the D0-brane, the linear term in (4.2) is cancelled and so the moduli space is not lifted, but we still expect that if the D0 tries to get fat inside the D4-brane, conformal symmetry of R^4 will be broken and size will not be a modulus of the instanton.

4.1 Chiral symmetry breaking and five dimensional Chern-Simons theory on a boundary

Let us put a boundary on the 4+1 dimensional system. The anomaly equation in 3+1 dimensions

$$\partial^0 J_0^5 = F_{01} F_{23} \tag{4.3}$$

as in 1+1 dimensions has an interpretation as coming from a 5d bulk current J^5 transverse to the 3+1 boundary. Again in the 3+1 theory it appears as though charge in created from no where, but it is really coming in from the 5th dimension. Charge can leave the left-hand-side three dimensional wall and enter the right hand side 3-wall. This is precisely the Goldstone-Wilczek current [35].

In the brane set-up, to add a boundary, we add NS5-branes so that the D4-brane can end on them, creating a 3+1 wall. The exact set-up is NS5-branes in directions 012345, N_c D4-brane in 01236, N_f D8-branes in 012345689. In order to satisfy the massive IIA equations of motion the NS5-branes must have D6-branes ending on them in directions 0123789. Now 6-4 strings are the massless 3+1 chiral fields on the boundary. The left-side has left-handed chiral superfields and the right-side has right-handed chiral superfields [36]. This set-up was discussed in [22]. However, if the 6-4 string representing a left-handed chiral superfield acquires some energy it can become a 8-4 string which in the D4-brane world volume theory looks like a massive five dimensional field. Then it can move to the other side of the interval and turn back into a 6-4 string becoming massless again. However, now it has become a right-handed chiral superfield. The superfield has changed chirality! But because of this big potential barrier proportional to the mass of the 5d field it is a rather unlikely event. It can become much more likely if we attach the string to a D0-brane. Since the D0-brane is electrically charged due to the 5d Chern-Simons term it wants to bind to the oppositely charged string making a BPS state. The heavy D0-brane can then carry a relatively light string across the interval. Moreover, the D0-brane has exactly the right potential to be an instanton effect.

$$V = e^{-r/L_6} = e^{-1/g_{YM}^2} \,. \tag{4.4}$$

where we have used the relation between the coupling of the D4-brane g_{YM} to the length of the interval L_6. We note here that this set-up is very much like the proposal of [2] where four-dimensional chiral fermions live on the boundary of the fifth dimension. The theory in five-dimensions is a Chern-Simons theory exactly as we have in our brane set-up here.

D0-brane is also similar to a baryon in that it is a composite of k quarks.

4.2 Confinement

There is an interesting mechanism for confinement in this picture. Consider massless 4-6 string at one edge of the D4-brane and an anti-4-6 string on the other edge. These represent quark anti-quark pairs. They are separated in the 5th dimension as well as in $\mathbb{R}^3$. As L_6 becomes of order the string scale, it is energetically favorable for the 6-4 string and the 4-6 string, if they are located in the same place in $\mathbb{R}^3$, to join into a 6-6 string. The 6-6 string is a gauge singlet under the color group $\mathrm{SU}(N_c)$ but a bifundamental under the flavor group $\mathrm{SU}(N_f) \times \mathrm{SU}(N_f)$. In this way, the 6-6 string is a meson.

5. Fun with 8-branes and 2-branes

5.1 D8-D2-D8 bound state

If we consider the potential for a D2-brane in the background of two D8-branes at positions $+A$ and $-A$, then the field theory on the D2-brane will be parity invariant with two massive fermion fields

$$\mathcal{L}_{D2} = (X + A)\bar{\psi}_+\psi_+ + (X - A)\bar{\psi}_-\psi_- \,. \tag{5.1}$$

From the supergravity we find that when one integrates out the fermions as well as the whole tower of string states, the potential exactly cancels between the 8-branes. The situation is then massless IIA on the inside of the walls and massive IIA on the outside. A D2-brane in the background of a flat massless IIA metric has no potential. The D2-branes sits on the top of a plateau with steep slopes on either side. The string theory smoothes out the sharp edges of the plateau.

6. Conclusions

It would be interesting to pursue the conjectured duality between the U(N) non-abelian Chern-Simons theory at level k and the fractional quantum Hall effect at level $\nu = k/M$ further. Also refining the picture of non-perturbative chiral symmetry breaking in the 3+1 theory on the boundary of the 4+1 Chern-Simons theory might be helpful in sorting out how much chiral symmetry breaking is related to confinement in real QCD. What is the relationship with the Chern-Simons solitons and particles we see in 3+1 dimensions? Are they related to baryons in any way? More speculatively can the fractional quantum Hall effect be related in any sensible way to chiral symmetry breaking in 3+1 dimensions?

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